

EVALUATION OF BEST-FIT FAILURE CRITERIA FOR TALCHER AREA.

Thesis submitted in partial fulfillment of the requirements for the degree of

Bachelor of Technology in Mining Engineering

By

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National Institute of Technology
Rourkela-769008
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Under the Guidance of

Dr. MANOJ KUMAR MISHRA



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CERTIFICATE

This is to certify that the thesis entitled "*EVALUATION OF BEST-FIT FAILURE CRITERIA FOR TALCHER AREA*" submitted by Mr. Akshay Aggarwal, Roll No. 110MN0507 in partial fulfillment of the requirements for the award of Bachelor of Technology degree in Mining Engineering at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by them under my supervision and guidance. To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

Date

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Abstract

Mining sector has always been a driving force in country's growth. Thermal energy is generated using coal as major raw material which has to be mined and handled before it could be used for power generation or in other industries. Extraction of mineral wealth from underground sources is filled with many uncertainties, rock characteristics is one of those. It is essential to predict the strength of rocks underlying and overlaying the coal so that safe mine could be designed. Various failure criteria have been developed to predict the nature of rock mass failure and its behavior. No failure criteria has been as yet designed specifically for Talcher area. Using the four existing failure criteria and six bore holes' data, a failure criteria has been developed for Talcher area. Mohr coulomb failure criteria proves to be least acceptable to predict the behavior of Talcher rock mass. Hoek Brown failure criteria, Yudhbir failure criteria and Ramamurthy failure criteria show promising results when analyzed with the actual data. Out of the four criteria, Ramamurthy failure criteria gives the least error in actual verses predicted values. The student t test also showed that it is acceptable.

Keywords: Failure criteria, best fit curve, least square error, student's t-test

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List of symbols and abbreviations

σ_1 = major principal stress (compressive stresses are taken as positive)

σ_2 = intermediate principal stress

σ_3 = minor principal stress

σ_n = normal stress

σ_t = uniaxial tensile strength of intact rock

τ = shear stress

ϕ = friction angle of intact rock or rock mass

c = cohesion of intact rock or rock mass

a and B = constants in Ramamurthy criterion

A , B and α = constants in Yudhbir criterion

m and s = material constant in the Hoek and Brown failure criterion

UCS = Uniaxial compressive strength

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Chapter 1: Introduction

This chapter deals with general over view of the project along with aim and objective of the investigation. The methodology adopted is also discussed briefly.

1.1 INTRODUCTION

Mining is the most important sector for development of any nation. It is a major source of power and raw material for almost all other industries in a country. Mining is the economic extraction of valuable minerals from earth for many purposes like generating power, pharmaceutical applications, infrastructures etc. It provided a base for the civilization to grow in all its form and acted as an example for the other sector of industries to breed. Mining is done in broadly two ways: underground extraction and surface/open pit extraction. It is essential to predict the behavior of the rock during the process of excavation. To predict the rock mass behavior, failure criteria are essential.

1.2 BACKGROUND OF THE PROBLEM

Mining sector has always been a driving force in country's growth. Thermal energy is the key source of power in our country. Thermal energy is generated using coal as major raw material which has to be mined and handled before it could be used for power generation or in other industries.

Hence it is essential to extract coal systematically and safely. Extraction of mineral wealth from underground sources is filled with many uncertainties, rock characteristics is one of those. It is essential to predict the strength of rocks underlying and overlaying the coal so that safe mine could be designed. It involves the determination and prediction of maximum bearing capacity of the rock/coal both in unconfined and confined states so as to design the different dimensions of excavation. In order to do so, it is desired to use a failure criteria to predict the stress and strength of the rocks. There exists no specific failure criteria for

Talcher area, MCL that is one of the richest coal resources of India. This project aims at developing a failure criteria specific to Talcher area by modifying the existing failure criterions.

1.3 AIM AND OBJECTIVE OF THIS STUDY

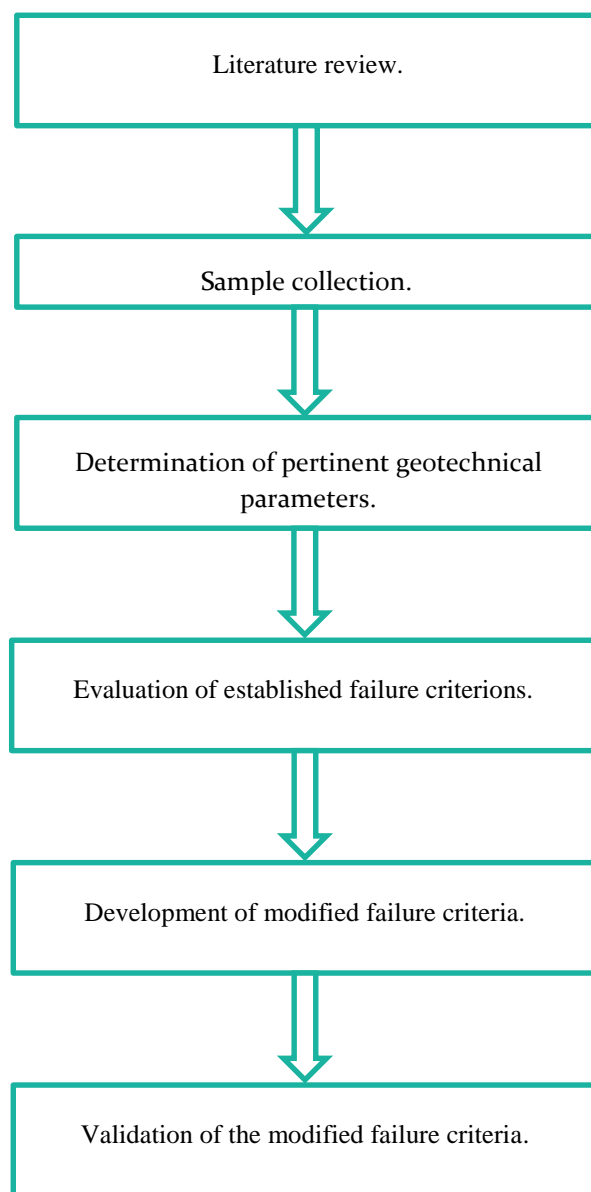
This project aims at finding out the best failure criteria for the rock mass at Talcher area by modifying the rock dependent constants of the existing intact rock failure criterions.

The goal was achieved by addressing the following objectives:

- Reviewing the existing failure criterions
- Evaluating their applicability for Talcher area.
- Modifying the existing failure criteria's constants based on bore-hole data of the Talcher formation.
- Finding out the best modified criteria for the formation.
- Finding out the confidence level of the failure criteria

1.4 METHODOLOGY

The aim and objectives are achieved by sampling, assaying, and laboratory testing the composite samples obtained from bore-hole drilling and then using mathematical statistics to find the best equation. The process followed to carry out this project is depicted in the flow diagram below step wise step.



The layout of the thesis consists of four chapters. Chapter 1 deals with basic layout of the project containing aim and objective of conducting the study along with the in-brief process adopted to carry out the project. Chapter 2 is the literature review containing description about the utility of the rock testing and failure criteria. It includes details about past studies carried out on similar base by other scientists and researchers. Chapter 3 contains details about the geology and lithology of the rock mass of Talcher on which the whole study is carried out. It also includes details about laboratory test done to find out the actual values of the stress and strength of the rock mass. Chapter 4 contains the results obtained from the test data along with the evaluation and analysis of failure criterions. It also consist of the discussion and conclusion derived from the investigation.

Chapter 2: Literature Review

This chapter explains the fundamental aspects of mining as well as summarizes published literature and articles related to failure criteria. A detailed study about rock, rock failure, need of failure and failure criterions is discussed.

2.1 ROCK AND ITS TESTING

The un-fractured blocks which exist between the structural discontinuities are known as rock material. This rock material is same for the intact rock. Intact rock consists of one or more variety of minerals. The intact rock pieces range from few millimeters to several meters depending on nature and type of rock material and existing discontinuities. In geology, the rock type is defined in accordance to the abundance, roughness and types of the minerals involved and in addition to mode of formation and degree of metamorphose, etc. On the basis of formation process, the rock is categorized in following classes: [16]

- igneous rocks (massive rocks of generally high strength),
- sedimentary rocks (softer minerals and often anisotropic rocks), and
- Metamorphic rocks, (great variety in structure, composition and properties).

In mining engineering practices, rock type is classified according to its rock quality parameters and potential mechanical performance. Therefore, the rock is described by its strength, stiffness, anisotropy, porosity, grain size and shape etc. The discontinuity is the collective term used for the whole range of mechanical defects such as joints, bedding planes, faults and fractures [1]. The term discontinuity does not consider the mode of origin of the feature and thus avoids any inferences concerning their geological origin. The mechanical behavior of the discontinuities is found to depend on the material properties of the intact rock itself, the joint geometry (roughness), the joint genesis (tension or shear joints) and the joint filling [2]. The definition of discontinuity here can be given as any significant mechanical break or fracture that has low shear strength, negligible tensile

strength and high fluid conductivity in comparison to the surrounding rock material [2]. Joint in the field of rock mechanics is used as a very general term and usually it is found to cover all types of structural weaknesses.

The term "rock mass" is defined as the rock material together along with the three-dimensional structure of discontinuities. Figure 2.1 describes the components of rock mass.

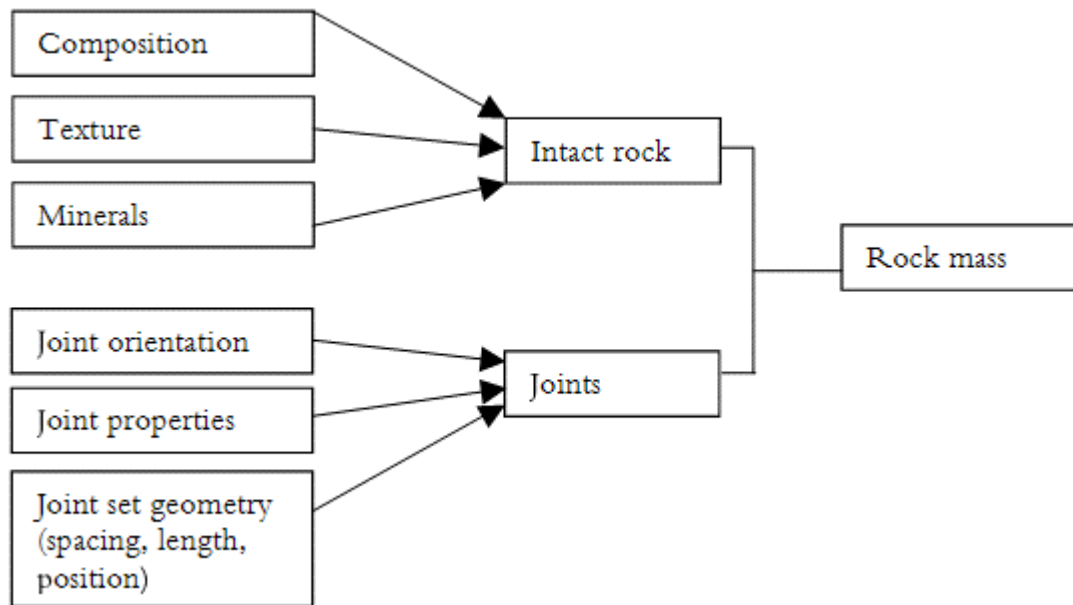


Figure 2.1: Definition of rock mass (Catrin Edelfbro, 2003)

Rock failure is defined as formation of faults and fracture planes, crushing, and relative motion of individual mineral grains and cements. Depending upon the failure characteristics, failure of solids can be divided into two groups: brittle or ductile, (Figure 2.2). In case of brittle failure sudden loss of strength is observed once the peak (σ_{peak}) has been reached. Despite of the fact that the rock may break, often a residual strength (σ_{res}) is observed, which is referred to maximum post-peak stress level that the material can sustain even after substantial deformation has taken place in the material (Brady & Brown, 1993).

The yield limit (σ_{limit}) is defined as the stress level at which departure from the elastic behavior is observed and the plastic deformation begins which is permanent in nature. For ductile failure it can be said that the loss of strength is not that sudden as observed in case of brittle behavior and there is a small, or no, strength reduction is observed after the yield limit is reached. Figure 2.2 describes the behavior of ductile failure.[3]

Failure of intact rock can often be categorized under as brittle failure. The harder igneous and some metamorphic rocks are often found to fail in a brittle manner. Weak sedimentary intact rocks tend to fail in a more ductile fashion.

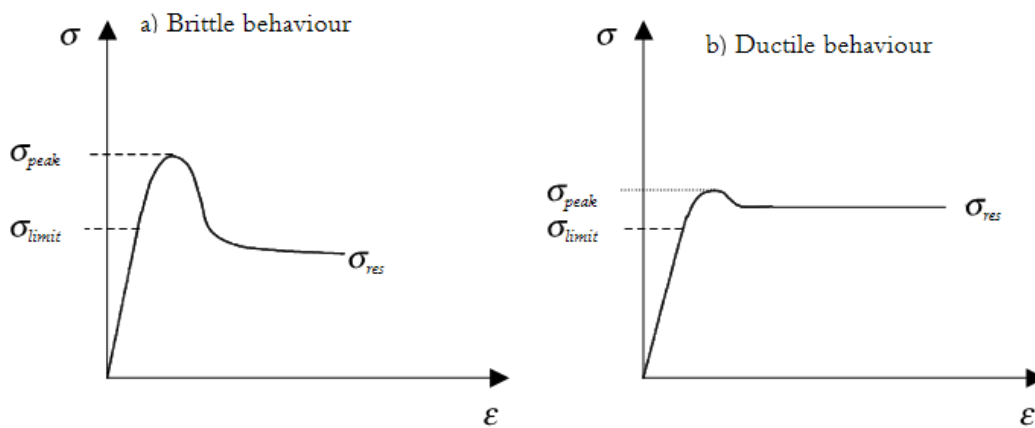


Figure 2.2 The strain/stress curves representing brittle and ductile failure, where σ_{limit} is the yield limit (stress), σ_{peak} is the peak stress and σ_{res} is the residual stress. (Catrin Edelbro, 2003)

When a particular combination of stress, strain, temperature and time exceeds a certain critical limit, failure of a rock mass occurs. The rock failure mechanism can be primarily categorized into 3 categories

- tensile failure,
- spalling (extensional failure), and

- shear failure.

Tensile failure occurs in the rock mass when the absolute value of the minor principal stress (σ_3) becomes less than the absolute value of the tensile strength of the rock mass (σ_{tm}). The tensile strength of discontinuities and rock masses is normally assumed to be zero.

Spalling is defined as fracturing of micro-defects parallel to the major principal stress and perpendicular to the minor principal stress. This causes extensional straining of the rock material in a direction parallel to the minimum principal stress [17].

Initial phase of the mechanism of shear failure of intact rock is similar to spalling. In shear failure, a shear zone is created and the confining stress present prevents the propagation of cracks along the major principal stress direction is prevented by. Shear displacement parallel to the orientation of the shear zone develops. Failure of the rock mass involves intact rock failure mechanisms as well as shear and dilation along existing discontinuities. Separation and rotation of blocks are also possible [17].

The combined strength of the intact rock and the various discontinuities in the rock mass, determines the strength of the rock masses. Instability of rock masses are very often characterized by:

- Block Failure-structurally controlled failure (loosening, block fall). Normally treated as a discontinuum problem.
- Failures that are induced from overstressing
 - Overstressing of massive rock (spalling, popping, strain burst) - normally treat as a continuum problem.

- Overstressing of jointed rock (shear failure, buckling) – can be treated both as a continuum and discontinuum problem.
 - Overstressing of granular materials (soils, heavily jointed rocks) – normally treated as a continuum problem.
- Instability because of faults and presence of weaken zones, Can be treated as a continuum or discontinuum problem, depending on the size of the weakness zone in relation to the construction size. For large scales, a fault or weakness zone can be treated as a joint and must therefore be analyzed as a discontinuum.

Since this project focuses on hard rock masses, continuum problems with failures induced from overstressing or instability in weakness zones in hard rock masses are most important [17].

2.2 UTILITY OF FAILURE CRITERIA

As the mining depth is increasing day by day the exact estimation of the rock mass strength is becoming more and more important. The stability problems that are found to occur due to deeper mining can be effectively reduced, by a better understanding of the rock mass strength. One of the most common and easiest way of determining the rock mass strength is by finding the failure criterion. The rock mass failure criteria which currently exists are stress dependent and they often include one or several parameters that describe the rock mass properties. These parameters are often based on classification or characterization systems. These criteria and systems that are used were selected based on the facts that they are published, well known, deemed suitable for underground excavations and/or instructive in India. Totally four failure criteria for intact rock are incorporated in this study [4].

Knowledge of the rock mass behavior in general, and the failure process and the strength in particular, is important for the design of drifts, ore passes, panel entries, tunnels and rock caverns. Mining methods based on caving and blocking of the ore, such as sublevel caving and block caving, also require knowledge of the rock mass strength. It is important to improve the design of the drifts, drilling and blasting, in order to decrease the costs. Furthermore, knowledge regarding the physical and mechanical properties of the rock mass is of great importance in order to reduce potential environmental disturbance from mining and tunneling. A better understanding of the failure process and a better rock mass strength prediction make it possible to, e.g., - reduce stability problems by improving design of the underground excavations, - improve near surface tunneling and ore extraction to avoid or minimize the area over which subsidence occurs due to tunneling and mining, and - reduce waste rock extraction. Despite the fact that research with focus on rock mass strength has been performed for at least the last 20 years, the mechanisms by which rock masses fail remain poorly understood. The behavior of the rock mass is very complex with deformations and sliding along discontinuities, combined with deformations and failure in the intact parts (blocks) of the rock mass. A mathematical description of the rock mass failure process is thus the need of time. Various failure criterions help in studying the rock failure trends and hence help in developing a safe and secure mine [17].

2.3 FAILURE CRITERION

All over the world various scientists have developed various failure criterions depending on different rock types and formations. One can never predict completely any underlying rock mass hence these failure criterions help in estimating the behaviors of rock mass.

The table below lists some of the failure criteria developed by various researchers all over the world.

Table2.1: List of existing failure criteria for intact rock [after Catrin Edelbro, 2003]

Failure equation	Development / comments	Author, criterion first published
$(\sigma_1 - \sigma_3)^2 = a + b(\sigma_1 + \sigma_3)$	An empirical generalisation of Griffith theory of intact rock.	Fairhurst (1964)
$\sigma_1 = \sigma_c + \sigma_3 + F\sigma_3^f$	Empirical test data fitting for intact rock.	Hobbs (1964)
$\sigma_1 = \sigma_c + a\sigma_3^b$	Not presented in detail in this report	Murrel (1965)
$\frac{\tau_m - \tau_o}{\sigma_c} = D \left \frac{\sigma_m}{\sigma_c} \right ^C$	Empirical curve fitting for intact rock. This version from Hoek is not presented in detail in this report	Hoek (1968)
$\sigma_1 = \sigma_c + a\sigma_3$	Triaxial tests on soft rock.	Bodonyi (1970)
$\sigma_1 = \sigma_3 + \sigma_c^{1-B} (\sigma_1 + \sigma_3)^B$	Empirical curve fitting for 500 rock specimens.	Franklin (1971)
$\sigma_1 = \sigma_3 + (m\sigma_c\sigma_3 + s\sigma_c^2)^{1/2}$	Application of Griffith theory and empirical curve fitting. Both for intact and heavily jointed rock masses	Hoek & Brown (1980)
$\frac{\sigma_1}{\sigma_c} = a + b \left(\frac{\sigma_3}{\sigma_c} \right)^\alpha$	Empirical curve fitting for 700 rock specimens. Both for intact and heavily jointed rock masses	Bieniawski (1974), Yudhbir et al., (1983)
$\sigma_1 = \sigma_3 + a\sigma_3 \left(\frac{\sigma_c}{\sigma_3} \right)^b$	Applied to 80 rock samples.	Ramamurthy et al., (1985)
$\sigma_1' = \left(\frac{M}{B} \sigma_3' + 1 \right)^B$	Empirical curve fitting for both soil and rock specimens.	Johnston (1985)
$\sigma_1 = \sigma_c \left(1 + \frac{\sigma_3}{\sigma_t} \right)^b$	Both for intact and heavily jointed rock masses	Balmer (1952), Sheorey et al., (1989)
$\sigma_1 = \sigma_3 + A\sigma_c \left(\frac{\sigma_3}{\sigma_c} - S \right)^{1/B}$	A, B and S are strength parameters	Yoshida (1990)

where: $\tau_m = (\sigma_1 - \sigma_3)/2$ and $\sigma_m = (\sigma_1 + \sigma_3)/2$

τ = shear stress, σ_1 = Major principal stress,

σ = Normal stress, σ_3 = Minor principal stress,

σ_c = Uniaxial compressive strength, σ_1' = Major normalized effective principal stress,

σ_t = Uniaxial tensile strength, σ_3' = Minor normalized effective principal stress,

and a, b, F, f, C, D, B, M and α are constants

For this project, four failure criteria have been exhaustively studied. Mohr Coloumb failure criteria and Hoek Brown failure criteria are widely accepted all over the world for most geological formations. Yudhbir failure criteria and Ramamurthy failure criterion were developed based on Indian Coal rock mass. These four failure criterions were used to evaluate the test data and to develop the best fit criterion for Talcher area.

2.4 MOHR COULOMB FAILURE CRITERIA FOR INTACT ROCK

Figure 2.3 shows tri-axial test results plotted in the form of a Mohr diagram. The Mohr circle is a very convenient way of plotting the principal stresses - the two principal stresses are plotted on the x-axis, and the radius of the circle is the maximum shear stress $(\sigma_1 - \sigma_3)/2$. The plot is therefore one of shear stress against normal stress. The Mohr circle plot can be used to determine stress magnitudes at different orientations [5].

As shown in Figure 2.3, the Mohr-Coulomb shear strength failure criterion is a linear envelope to the Mohr circles. The equation of the line is given by:

$$\tau = S_i + \sigma \tan \phi \quad (1)$$

Since rock is weak in tension, the criterion is incorrect to the left of the ordinate axis, and a tension cut off is usually used, as shown in Figure 2.3 [5].

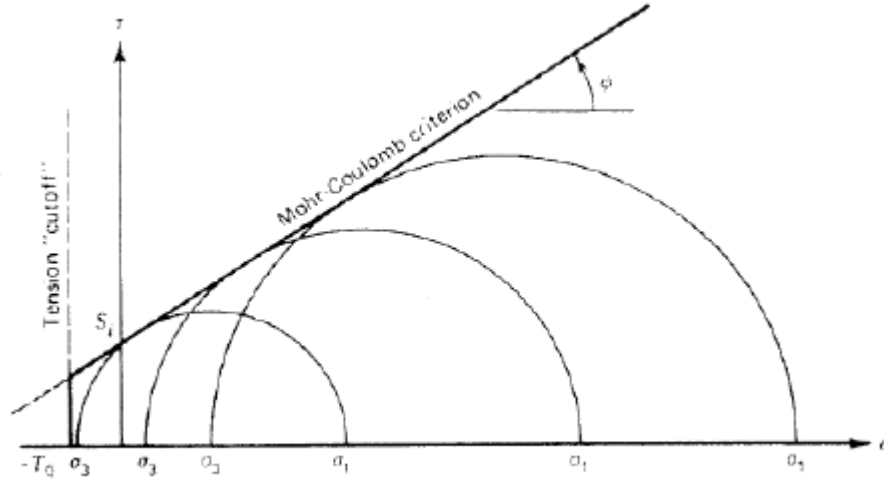


Figure 2.3 Mohr Coulomb Failure criteria (Goodman, 1980)

The Mohr-Coulomb criterion is also commonly used to represent the residual strength of the rock as shear failure continues to take place on the failure surface that has been created in the failure of the intact rock. The criterion is commonly written as:

$$\tau = C + \sigma_n \tan \phi \quad (2)$$

where C is the cohesion, and σ_n is the normal stress.

It can also be expressed in the form:

$$\sigma_1 = \frac{[2C \cos \phi + \sigma_3 (1 + \sin \phi)]}{(1 - \sin \phi)} \quad (3)$$

where C and ϕ are the cohesion and angle of friction respectively.

Since:

$$\sigma_c = \frac{2C \cos \phi}{(1 - \sin \phi)} \quad (4)$$

then equation (3) can be rewritten as:

$$\frac{\sigma_1}{\sigma_c} = 1 + C \frac{\sigma_3}{\sigma_c} \quad (5)$$

Where $C = \frac{(1 + \sin \phi)}{(1 - \sin \phi)}$ and σ_c is the UCS.

The orientation of the predicted shear failure plane is $(45 + \phi/2)$ degrees, where this angle is measured in the $(\sigma_1 - \sigma_3)$ space from the σ_3 axis.

Limitations

The Mohr-Coulomb criterion is not a particularly satisfactory criterion for rock, since:

- it implies that a major shear fracture occurs at peak strength. The criterion is likely to give incorrect results if the failure mechanism is not shear.
- it implies a direction of shear failure which often does not agree with observations, particularly in brittle rock;
- it is linear and peak strength envelopes determined experimentally are usually non-linear, as shown in Figure 2.4 below.
- it assumes that friction and cohesion are acting in unison
- it will be noticed that only σ_1 and σ_3 are used and that σ_2 is ignored in this criterion [5].

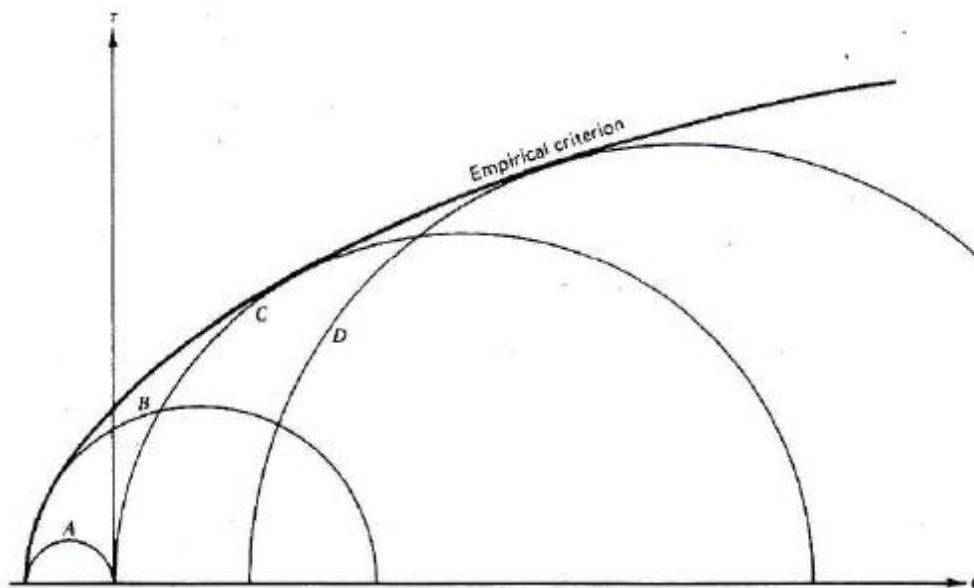


Figure 2.4: Non Linear Mohr Coulomb Failure envelope (Goodman, 1980)

2.5 HOEK BROWN FAILURE CRITERIA

Hoek and Brown developed an empirical criterion for rock and rock mass failure based on tests on intact rock and on rock mass models [6]. The generalized form of the Hoek-Brown failure criterion is:

$$\sigma_1 = \sigma_3 + \sigma_c \left(m \frac{\sigma_3}{\sigma_c} + s \right)^a \quad (6)$$

where m is the Hoek-Brown constant for the rock mass, s and a are constants which depend on the rock mass characteristics. σ_c is the UCS of the intact rock [7].

For intact rock, the above equation simplifies to

$$\sigma_1 = \sigma_3 + \sigma_c \left[m \frac{\sigma_3}{\sigma_c} + s \right]^{0.5} \quad (7)$$

Limitations

- The Hoek-Brown criterion is a shear based criterion and therefore has similar limitations to the Mohr-Coulomb criterion.
- Only σ_1 and σ_3 are used and σ_2 is ignored.
- It only applies to the "central" range of rock masses, i.e. well-jointed rock masses in which the joints control behavior rather than the rock material or individual significant planes of weakness.
- Failure initially develops as damage of the rock, followed by spalling failure and then ultimately transition to shear failure in brittle rocks. The criterion has recently been modified to cater for both high strength brittle rock conditions and low strength weak ground conditions. The modified relationships are shown below [6].

2.6 RAMAMURTHY FAILURE CRITERIA

Laboratory studies on jointed plaster and sandstone specimens formed the basis and provided input data for the development of the Ramamurthy criterion. This criterion is only related to its own classification system that represents the rock mass strength reducing parameter. This criterion is developed based on uniaxial compression and triaxial tests for more than 100 different rock types [8].

$$\sigma_1 = \sigma_3 + B\sigma_3 \left[\frac{\sigma_c}{\sigma_3} \right]^a \quad (8)$$

B and a, are constants gained by triaxial tests [8].

2.7 YUDHBIR FAILURE CRITERIA

This criterion is based on triaxial tests carried out on 20 trial crushed and intact model material samples, uniaxial tests, direct shear tests and Brazilian tests [8].

$$\frac{\sigma_1}{\sigma_c} = A + B \left[\frac{\sigma_3}{\sigma_c} \right]^\alpha \quad (9)$$

In this criterion, α is a constant parameter and is independent of rock type. And, its suggested value is 0.65 [8]. The suggested values for B depends on rock type and is a rock material constant [8]. Also, A is a dimensionless parameter whose values depend on rock type [8]. The value of B as defined by Bieniawski-Yudhbir for different rocks are given in table below:

Table 2.2: Value of B in Bieniawski-Yudhbir criteria

Rock Type	Value of B
Tuff shale, limestone	2
Siltstone, mudstone	3
Quartzite, sandstone, dolerite	4
Norite, granite, quartzdiorite, chert	5

Seven different failure criteria by comparing them to published polyaxial test data ($s_1 > s_2 > s_3$) for five different rock types at a variety of stress states. They found that the polyaxial criteria Modified Wiebols and Cook and Modified Lade achieved a good fit to most of the test data. This is especially true for rocks with a highly s_2 -dependent failure behavior (e.g. Dunham dolomite, Solenhofen limestone). However, for some rock types (e.g. Shirahama Sandstone, Yuubari shale), the intermediate stress hardly affects failure and the Mohr–Coulomb and Hoek and Brown criteria fit these test data equally well, or even better, than the more complicated polyaxial criteria. The values of C_0 yielded by the Inscribed and the Circumscribed Drucker–Prager criteria bounded the C_0 value obtained using the Mohr–Coulomb criterion as expected. In general, the Drucker–Prager failure criterion did not accurately indicate the value of s_1 at failure. The value of the misfits achieved with the empirical 1967 and 1971 Mogi criteria were generally in between those obtained using the triaxial and the polyaxial criteria. The disadvantage of these failure criteria is that they cannot be related to strength parameters such as C_0 : It was also found that if only data from triaxial tests are available, it is possible to incorporate the influence of σ_2 on failure by using a polyaxial failure criterion [9].

Four different rock failure criteria were compared based on triaxial test data of ten different rock strength data using various statistical methods. Least square, least median-square and re-weighted least square techniques were used to determine the best fit parameters utilizing the experimental data that describes the failure state for each criterion. The least median square method could identify the scattered data and these scattered data points were observed at higher confining stress. It was observed that the fitting of failure criteria to

different rock strength data depends upon the statistical methods used. The prediction of unconfined compressive strength and failure strength for different rocks estimated using various statistical methods are discussed in terms of different statistical performances of the prediction [10].

Linear Mogi criterion does a good job in representing rock failure under polyaxial stress states. When $\sigma_2 = \sigma_3$; the linear version of Mogi's triaxial failure criterion reduces exactly to the Coulomb criterion. Hence, the linear Mogi criterion can be thought of as a natural extension of the Coulomb criterion into three dimensions (i.e., polyaxial stress space). As Mohr's extension of the Coulomb criterion into three dimensions is often referred to as the Mohr–Coulomb criterion, it was proposed that the linear version of the Mogi criterion be known as the “Mogi–Coulomb” failure criterion. Hence it was concluded that the classical Coulomb failure criterion can be thought of as a special case, which applies only when $\sigma_2 = \sigma_3$; of the more general linear Mogi failure criterion [11].

Five rock failure criteria for intact salt rock were evaluated to find the best fit criteria. Full scale comparison of all criteria for 3 rock types was conducted based on five stand statistics calculated from least square curve fitting. The results indicated that all non-linear criteria with a basic power form are efficient in predicting the strength trends in the low tension area as well as in the high compression area of the soft rocks. The generalized Hoek Brown criteria is proven to perform best in two rock strength data followed by one for the Bieniawski empirical criteria [12].

Rigorous statistical analysis on three failure criteria was conducted to find fitting failure criteria to laboratory strength tests. The test data carried out on four different types of samples- Dunham dolomite, Indiana limestone, norite and sandstone, was used. It was concluded that forward extrapolation of any linear criterion into the range of triaxial stress states is unlikely to produce reliable strength estimates. Backward extrapolation from linear fits to confining pressure test data is also unreliable. Hence a non-linear failure criteria is needed to overcome the short coming of linear failure criteria [13].

Chapter 3: Materials and Methods

This chapter deals with the bore hole core collection, preparation and testing of rock samples.

3.1 GEOLOGY OF FORMATION

The cored samples were obtained from Talcher coalfield area, MCL. Talcher rock mass is located at about 150kms from the city Bhubneshwar, the capital of Orissa state in India. The Talcher area is extended over an area of 1813 sq km. The latitudinal extent stretches from 20°50' N to 21°15' N and the longitudinal extent stretches from 84°09' E to 85°33' E. It covers the districts of Dhenkanal, Angul and Sambalpur. The latitudinal extent of the region covered is 20° 55' 00'' N to 21° N and the longitudinal extent is from 85°05' E to 85°10' E.

The reserve in this region belonging to Lower Permian age have formations named Barakar, Karharbari and Talchir. The Barakar formation has a lithology consisting of Medium to coarse-grained sandstones, shales, coal seams with oligomictic conglomerate at base having average depth of 500m. The Karharbari formation has lithology consisting of medium to coarse-grained sandstones, shales and coal seams having an average depth of 270m. The Talchir formation has lithology consisting of diamictite, fine to medium-grained greenish sandstone, shales, rhythmite, turbidite etc. found at an average depth of 170m. The reserve belonging to upper Permian age has formation named Barren Measures consisting of greenish grey to buff colored pebbly, coarse to medium grained highly ferruginous sandstone. The reserve in this region belonging to recent upper Permian to Triassic age has formation named Kamthi consisting of Alluvium, laterite, fine to medium-grained sandstone, carbonaceous shale, coal bands, with greenish sandstone, pink clays and pebbly sandstones at top having average depth of 250m.

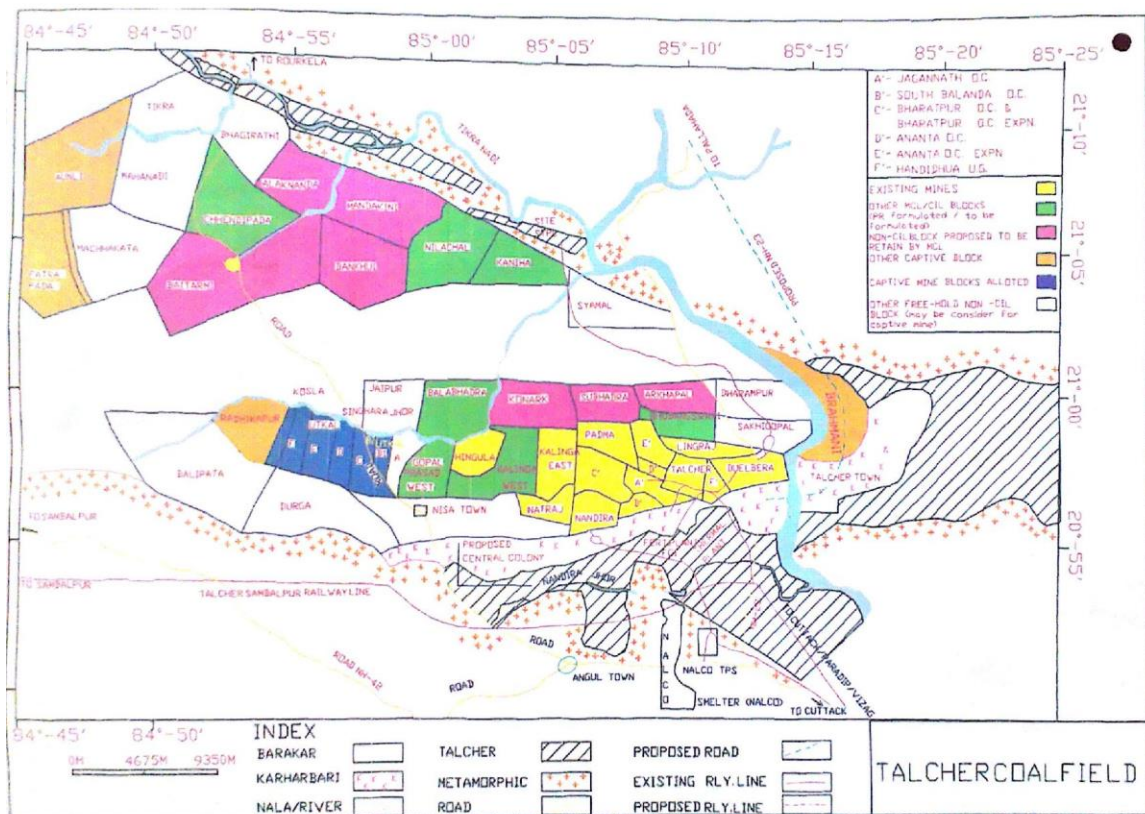


Figure 3.1: Geological Map of Talcher area

The classification of strata above Barakar Formation in the Talcher coalfield has undergone rather rapid modification. The Supra-Barakar Strata were sub-divided into Raniganj, Panchet and Mahadeva Formations in late sixties and early seventies (GSI, 1971).

3.2 METHOD OF SAMPLE COLLECTION AND PROCEDURE

Six bore-holes were drilled over a span of 1sq km area. The bore-holes had an average diameter of 72-74 mm and their depth varied from 70m to 141m deep. Core preparation was done based on IS 9179-1979 standard. The length by diameter ratio of the composite was maintained at 2 to 2.5. The core sample contained coal, grey shale, shaly sandstone coarse sandstone, medium sandstone and fine grained sandstone. After the core was

extracted from the bore-hole, the 2m length samples were kept in a wooden box in successive continuation. The box was sealed to prevent the sample from losing the in-situ moisture. Then, the wooden boxes were transported to the laboratory ensuring that samples do not get damaged during transportation. The vibration resistant arrangements were used to secure the boxes.



Figure 3.2: Bore-hole samples stored in a wooden box for transportation to laboratory

The boxes were opened right before the test. The core samples were cut as per the size required, polished and grinded and moisture content was measured. Thereafter the compressive strength test was carried out to find out the UCS value. The tensile test was conducted to calculate the axial stress and confining stress values. Triaxial test was conducted to find out the C and ϕ value of each composite sample. Compressive strength was computed based on IS 9143-1979 (Reaffirmed 1996) standard. The tensile strength was computed based on IS 10082-1981 (Reaffirmed 1996) standards. The triaxial test was carried out as per IS 13047-1991 (Reaffirmed 2001) standard. In triaxial test, with oil or

water as confining medium, the confining pressures are equal in all directions (i.e. in terms of principal stresses: for a compression test $\sigma_1 \neq \sigma_2 = \sigma_3$ and for tensile: $\sigma_1 = \sigma_2 \neq \sigma_3$). The actual values of σ_1 , σ_2 , σ_3 , ϕ and C were hence obtained.



Figure 3.3: UCS Testing Machine

After conducting the above laboratory tests, a dataset of 110 readings was obtained. Two to three same samples were tested to obtain one reading. Out of these 110 readings, 70 readings were randomly selected to find out the constants of the failure criteria equations and to modify the existing failure criteria and the remaining 40 reading were later used to evaluate the hence formed modified failure criteria equations. Table 3.1 shows the data obtained from the tests conducted.

Table 3.1: Data Range obtained from the laboratory test.

S. No.	Type of Rock	σ_c (MPa)	Triaxial Strength Parameters			
			σ_1 (MPa)	σ_3 (MPa)	ϕ (degree)	C(MPa)
1	Coarse Grain Sandstone	10.17-13.79	23.25-40.46	3.922-7.845	33-35	3.8
2	Coarse Sandstone	6.87-21.42	15.17-42.17	3.922-7.845	20-37	2.4-5.4
3	Medium Coarse Sandstone	6.845-14.21	20.44-46.4	3.922-7.845	22-33	3.6-5
4	Medium Sandstone	9.26-16.44	21.29-54.67	3.922-7.845	23-38	3.6-4.6
5	Grained sandstone	13.14	28	3.922-7.845	34	3.9
6	Coal	13.42-33.76	18.94-61.74	3.922-7.845	19-40	5.1-10.8
7	Fine Sandstone	13.59-34.82	27.19-70.4	3.922-7.845	28-36	3.6-5.2
8	Coarse Medium Sandstone	15.38	37.9	3.922-7.845	34	4.2
9	Grey Shale	23.98	41.64	3.922-7.845	34	6.4
10	Fine Shaly Sandstone	28.88	44.57-56.07	3.922-7.845	35	8
11	Shaly Sandstone	29.24-33.51	44.2-61.89	3.922-7.845	38	7.02

The failure criteria aims at developing a model to find out σ_1 value depending on rest parameters of the rock. The equations were hence rearranged to represent in a similar form. Using the laboratory test data, experimental relations were computed by plotting the data and finding out the best fit equation. The best fit equations were then compared with the failure criteria to find the rock dependent constants. Similar process was followed for Mohr Coulomb failure criteria, Hoek-Brown failure criteria, Yudhbir failure criteria and Ramamurthy failure criteria. The modified equations were thus obtained. Now these equations were used to calculate the value of σ_1 taking in other parameters from remaining 40 test data. The predicted σ_1 and actual σ_1 were plotted against each other to observe the variation. The intercept of best fit equation of these plots was made zero so as to see the

deviation of slope of best fit curve from unity. The regression coefficient of best fit equations of these plots (between predicted σ_1 and actual σ_1) were also compared. Then least square error test was conducted for all four failure criteria and the equation having least error was accepted. The method of least squares is a standard approach to the approximate solution of over determined systems, i.e., sets of equations in which there are more equations than unknowns. "Least squares" means that the overall solution minimizes the sum of the squares of the errors made in the results of every single equation. The best fit in the least-squares sense minimizes the sum of squared residuals. A residual is the difference between an observed value and the fitted value provided by a model [15].

Student's t -test was conducted to validate the null hypothesis taken after least square error test. A t -test is any statistical hypothesis test in which the test statistic follows a Student's t distribution if the null hypothesis is supported. It can be used to determine if two sets of data are significantly different from each other, and is most commonly applied when the test statistic would follow a normal distribution if the value of a scaling term in the test statistic were known [16].

Chapter 4: Analysis and Results

This chapter deals with the rock test results and the interpretation of these in existing failure criteria.

4.1 EVALUATION OF EXISING CRITERIA

The data was used to analyze the existing four criterions and to modify those to fit the physical data of obtained from laboratory tests.

4.1.1 Mohr Coulomb criteria[5]

According to Mohr Coulomb criteria, failure criteria is:

$$\frac{\sigma_1}{\sigma_c} = 1 + C \frac{\sigma_3}{\sigma_c} \quad (5)$$

which can be written as:

$$\sigma_1 = (\sigma_3.C) + \sigma_c \quad (10)$$

using σ_c and C values from data, σ_1 was plotted against σ_3 . The graph (Figure 4.1) was obtained.

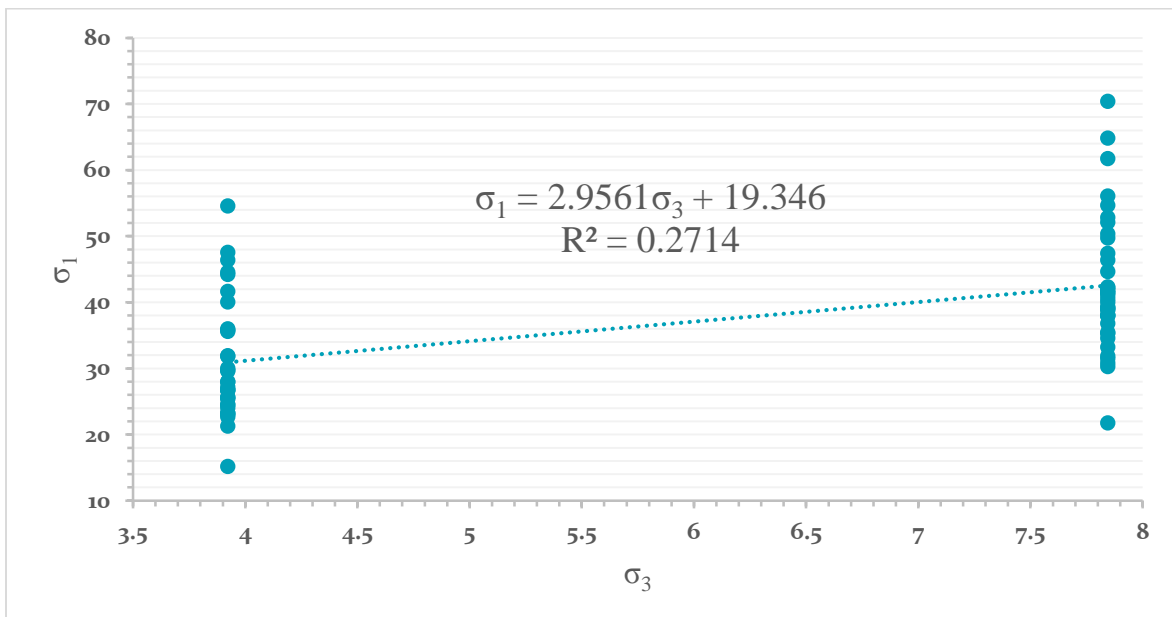


Figure4.1: Mohr Coulomb evaluation of equation

The triaxial test was conducted with fixed confinement that resulted in variable σ_1 for all samples. The triaxial loading ranges between 14MPa to 56MPa when the confinement was

3.9MPa. Similarly when the loading was increased to a range between 22MPa to 52MPa when the confinement was 7.8MPa.

Using Figure 4.1 plot, a comparison was made between the liner best fit equation and the actual Mohr Coulomb equation. On comparison following values of constants were observed:

$$\sigma_c = 19.346 \text{ and } C = 2.956103$$

With a Regression coefficient of 0.2714

4.1.2 Hoek Brown Criteria [6]

According to Hoek Brown criteria, failure criteria is:

$$\sigma_1 = \sigma_3 + \sigma_c \left[m \frac{\sigma_3}{\sigma_c} + s \right]^{0.5} \quad (7)$$

which can be written as:

$$\left[\frac{(\sigma_1 - \sigma_3)}{\sigma_c} \right]^2 = m \frac{\sigma_3}{\sigma_c} + s \quad (11)$$

using σ_1 , σ_3 and σ_c values from data, $\left[\frac{(\sigma_1 - \sigma_3)}{\sigma_c} \right]^2$ was plotted against $\frac{\sigma_3}{\sigma_c}$. The graph (Figure 4.2) was obtained.

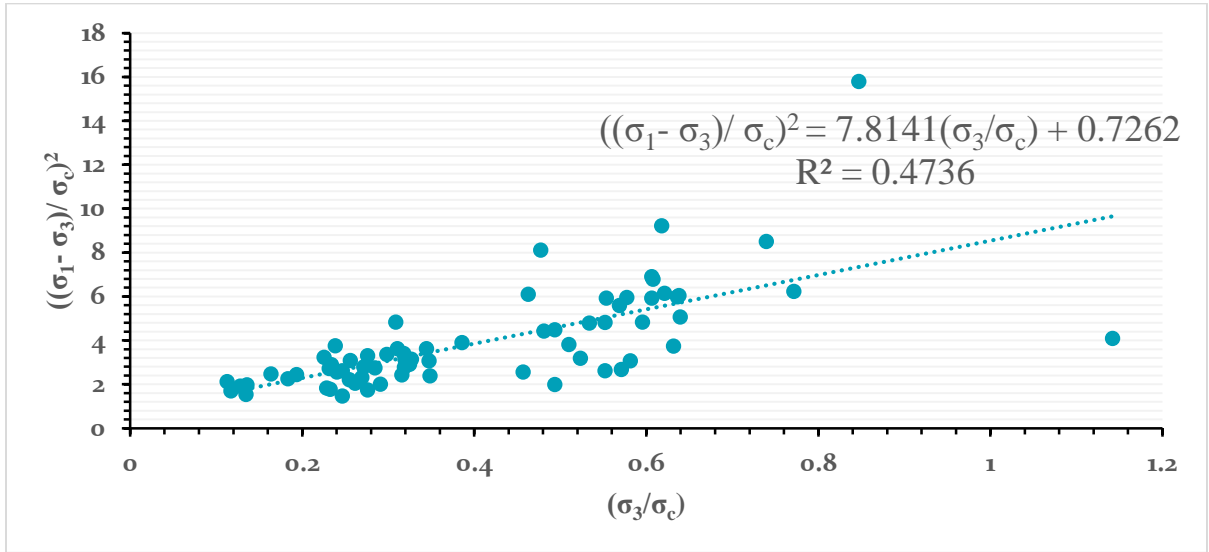


Figure 4.2: Hoek Brown evaluation of equation

The plot (Figure 4.2) shows that there is a cluster of readings obtained between 0.1 to 0.4 of σ_3/σ_c . The readings got more scattered later on. The regression coefficient here increased to 0.4736 which is greater than one obtained when using Mohr-Coulomb criteria.

Using Figure 4.2 plot, a comparison was made between the liner best fit equation and the actual Hoek Brown equation. On comparison following values of constants were observed:

$$m = 7.8141$$

$$s = 0.7262$$

4.1.3 Ramamurthy Criteria [7]

According to Ramamurthy criteria, failure criteria is:

$$\sigma_1 = \sigma_3 + B\sigma_3 \left[\frac{\sigma_c}{\sigma_3} \right]^a \quad (8)$$

which can be written as:

$$\log \left[\frac{(\sigma_1 - \sigma_3)}{\sigma_3} \right] = B \log \left[\frac{\sigma_c}{\sigma_3} \right] + \log a \quad (12)$$

using σ_1 , σ_3 and σ_c values from data, $\log \left[\frac{(\sigma_1 - \sigma_3)}{\sigma_3} \right]$ was plotted against $\log \left[\frac{\sigma_c}{\sigma_3} \right]$. The graph (Figure 4.3) was obtained.

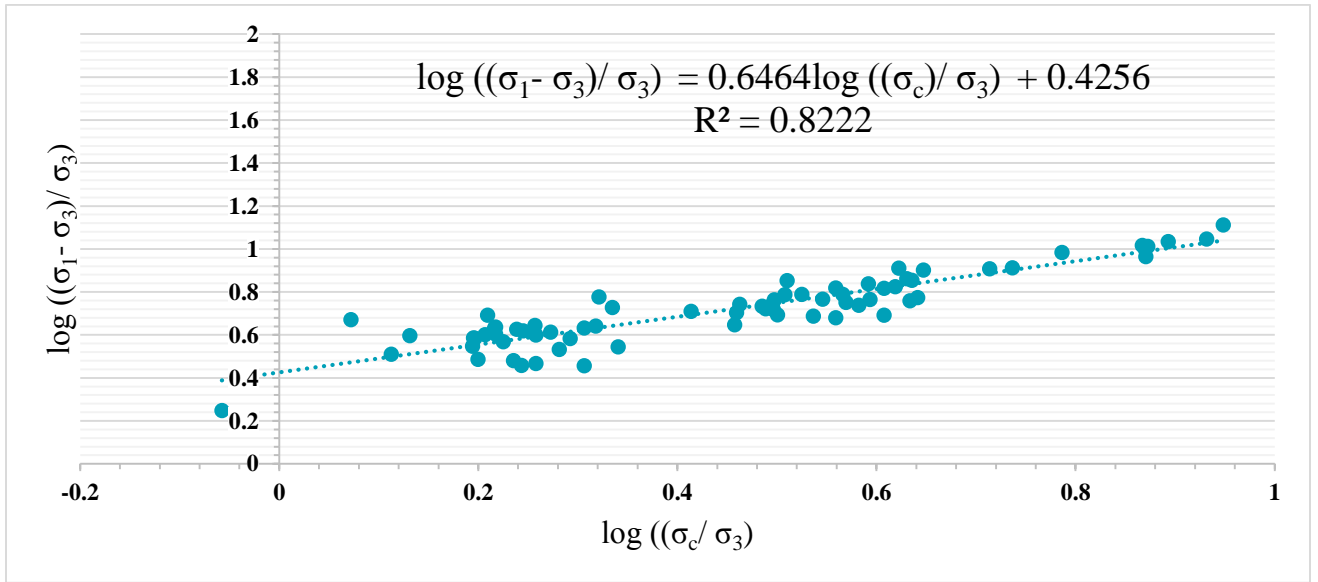


Figure 4.3: Ramamurthy evaluation of equation

The plot (Figure 4.3) shows that the cluster of readings is in two zones. One lying between 0.2 to 0.4 and another between 0.4 to 0.6 of $\log \left(\frac{\sigma_c}{\sigma_3} \right)$. The plot also shows that the variation in data is not huge and majority of the readings are close to the best fit linear curve. The regression coefficient obtained here is 0.822 which is almost double of the one obtained in Hoek Brown criteria.

Using Figure 4.1 plot, a comparison was made between the linear best fit equation and the actual Ramamurthy equation. On comparison following values of constants were observed:

$a = 2.6644$

$B = 0.6464$

4.1.4 Yudhbir Criteria [8]

According to Yudhbir criteria, failure criteria is:

$$\frac{\sigma_1}{\sigma_c} = A + B \left[\frac{\sigma_3}{\sigma_c} \right]^\alpha \quad (9)$$

which can be written as:

$$\log \left[\frac{(\sigma_1 - \sigma_c)}{\sigma_c} \right] = a \log \left[\frac{\sigma_3}{\sigma_c} \right] + \log B \quad (13)$$

using σ_1 , σ_3 and σ_c values from data, $\log \left[\frac{(\sigma_1 - \sigma_c)}{\sigma_c} \right]$ was plotted against $\log \left[\frac{\sigma_3}{\sigma_c} \right]$. The graph

(Figure 4.4) was obtained:

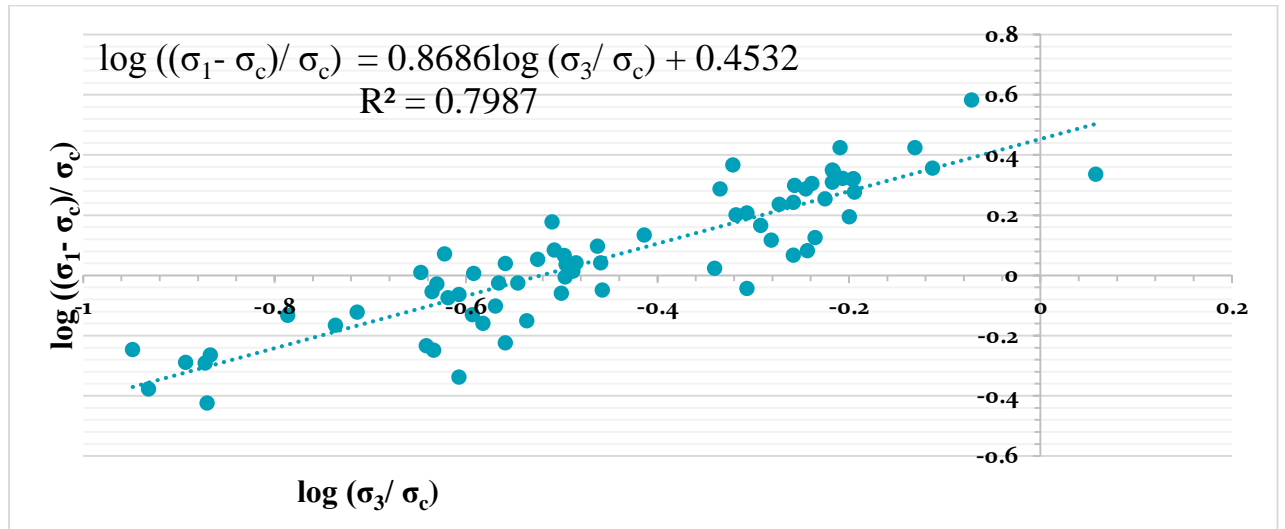


Figure 4.4: Yudhbir evaluation of equation

The plot (Figure 4.4) shows that the variation in data is more than that seen in Ramamurthy criteria. This resulted in slight decrease in regression coefficient which here is 0.7987.

Using Figure 4.1 plot, a comparison was made between the liner best fit equation and the actual Yudhbir equation. On comparison following values of constants were observed:

$$a = 0.8686$$

$$B = 2.83923$$

4.2 ANALYSIS

Using best fit equations of all the plots obtained by using the 4 failure criteria and data, values of constants of each equation were found and hence the system of modified equation was designed. The modified equations are as below (Table 4.1):

Table 4.1: Modified developed equations from existing failure criteria

S. No	Failure Criteria	Equation obtained	Regression Coefficient
1	Mohr Coulomb	$\sigma_1 = 19.346 + 2.956103 \sigma_3$	0.2714
2	Hoek Brown	$\sigma_1 = \sigma_3 + \sigma_c \left[7.8141 \frac{\sigma_3}{\sigma_c} + 0.73 \right]^{0.5}$	0.4736
3	Ramamurthy	$\sigma_1 = \sigma_3 + 0.6464 \sigma_3 \left[\frac{\sigma_c}{\sigma_3} \right]^{2.6644}$	0.8222
4	Yudhbir	$\frac{\sigma_1}{\sigma_c} = 1 + 2.83923 \left[\frac{\sigma_3}{\sigma_c} \right]^{0.8686}$	0.7987

These equations were developed using intact test results of 70 sample types. These developed equations were evaluated to predict the axial failure load with the existing σ_c , σ_3 and C of the other laboratory test results. A graph was plotted between $\sigma_{1(\text{predicted})}$ and $\sigma_{1(\text{actual})}$. Ideally the slope of the obtained equation should be equal to one with a Regression coefficient of 100%. The equation having values closed to ideal value can be considered best for the data. Following plots were obtained (Figure 4.5, 4.6, 4.7 and 4.8)

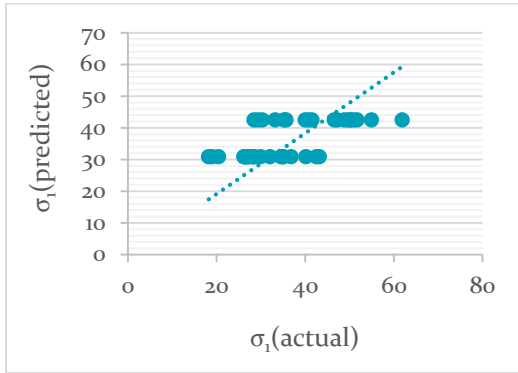


Figure 4.5: Plot between σ_1 (predicted) verses σ_1 (actual) based on modified Mohr Coulomb criteria

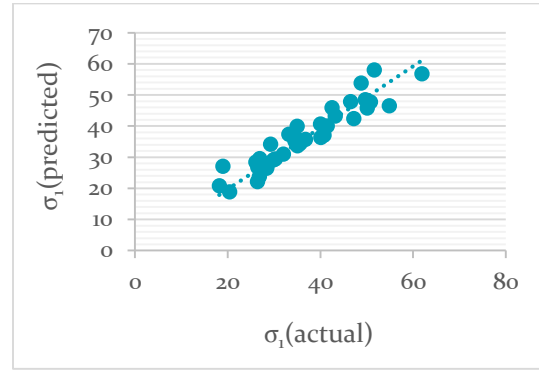


Figure 4.7: Plot between σ_1 (predicted) verses σ_1 (actual) based on modified Ramamurthy criteria

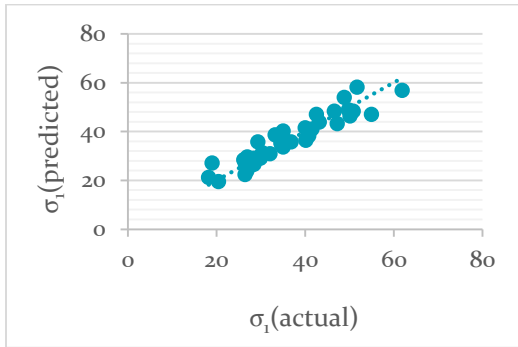


Figure 4.6: Plot between σ_1 (predicted) verses σ_1 (actual) based on modified Hoek Brown criteria

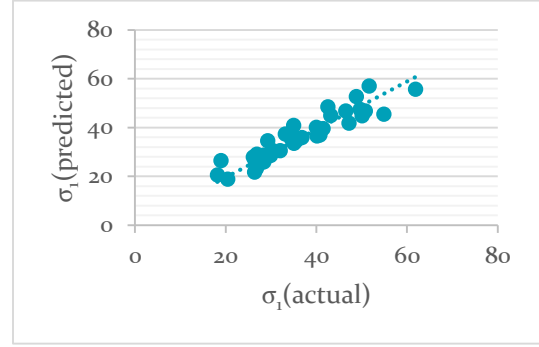


Figure 4.8: Plot between σ_1 (predicted) verses σ_1 (actual) based on modified Yudhbir criteria

The mutual relation between developed and actual axial loading are shown in Figures 4.5, 4.6, 4.7 and 4.8. The Ramamurthy approach gives exhibits best relation with the regression coefficient of 0.8842. The summary of all analysis up till now is presented in table below.

Table 4.2: Compilation of results of analysis of developed equation

Failure Criteria	Regression coefficient during finding coefficients	Regression coefficient of the σ_1 (predicted) versus σ_1 (actual) plot	Deviation of slope of σ_1 (predicted) versus σ_1 (actual) plot from 1
Mohr Coulomb	0.2714	-0.881	0.0422
Hoek Brown	0.4736	0.88	0.0022
Ramamurthy	0.8222	0.8842	0.0136
Yudhbir	0.7987	0.8665	0.0195

It is evident from above table (Table 4.2) that out of 4 criteria, Ramamurthy proves to fit best out of the above four equations for the data obtained from Talcher rock mass. In case of deviation from slope of $\sigma_1(\text{predicted})$ versus $\sigma_1(\text{actual})$ plot from 1, Hoek brown equation give the minimum result. The negative regression coefficient of Mohr Coulomb equation shows that the value of $\sigma_1(\text{predicted})$ varies inversely with $\sigma_1(\text{actual})$. To validate if Ramamurthy equation is best for the given data, least square test was conducted. The best fit in the least-squares sense minimizes the sum of squared residuals, a residual being the difference between an observed value and the fitted value provided by a model. Difference of $\sigma_1(\text{predicted})$ and $\sigma_1(\text{actual})$ was taken and squared. The sum of respective squared difference is the least square error.

Following was observed after carrying out least square test:

Table 4.3: Least square error test results

Failure Criteria	Regression coefficient during finding coefficients (in %)	Regression coefficient of the $\sigma_1(\text{predicted})$ versus $\sigma_1(\text{actual})$ plot (in %)	Deviation of slope of $\sigma_1(\text{predicted})$ versus $\sigma_1(\text{actual})$ plot from 1	Least square error
Mohr Coulomb	27.14	88.1	0.5098	65.1024
Hoek Brown	47.36	88	0.0022	23.47
Ramamurthy	82.22	88.42	0.0136	11.45
Yudhbir	79.87	86.65	0.0195	13.21

The least square error analysis shows that Mohr Coulomb has the maximum error i.e 65.1024. Ramamurthy approach gives the least error of 11.45. It can thereby be said that Ramamurthy best fits to judge the rock mass characteristics at Talcher area.

In order to justify our null hypothesis taken above, student's t test was conducted. This test is used to compare the means of two samples (or treatments), even if they have different numbers of

replicates. In simple terms, the t -test compares the actual difference between two means in relation to the variation in the data (expressed as the standard deviation of the difference between the means).

Table 4.4: Student's t -test result

Failure Criterion	t value	Regression coefficient of result by chance (%)
Ramamurthy	-0.141583748	0-25

The validation of Ramamurthy approach was confirmed with the Student's t -test. The t value obtained was -0.14158 showing probability of wrong result being less than 25.

4.3 CONCLUSION

The investigation considered about 110 rock sample test results. The data were used to determine the best fit failure criteria and evaluated further. The following conclusions are obtained from the investigation.

- Only 8 coal samples were found out of 110 data of 6 bore-holes.
- The roof contains coarse, fine and medium grained sandstone and shale.
- The rock material of the area is moderately strong.
- Mohr Coulomb criterion had least correlation and Ramamurthy criterion had maximum correlation.
- Modified Ramamurthy equation exhibits the best fit failure criterion with regression coefficient of 0.8222.
- The t test result validate the modified Ramamurthy criterion.

4.4 Future Scope

To further this project following studies can be carried out:

- Use more bore-hole samples to obtain a data set bigger than this to have more consistency in calculating the constants of the failure criterions.
- Use the C and ϕ values from the data set to validate the equations developed.

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